KUMON

J 11 a

Factorization 1

Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	1	2	3	4~

Formula $x^2 + (a+b)x + ab = (x+a)(x+b)$

Factor the following quadratic expressions, by applying the above formula. (Note that this method was introduced in Level I.)

(1) $x^2 + 8x + 15 =$

(2) $x^2 + 10x + 21 =$

(3) $a^2 - 7a + 10 =$

(4) $a^2 + 3ab - 10b^2 =$

(5) $12x^2 + 24x - 96 = 12(\qquad\qquad)$

$=$

(6) $ax^2 - 2ax - 8a =$

J 11 b

Ex. $(x+y)^2-3(x+y)-10$

$=[(x+y)-5][(x+y)+2]$

$=(x+y-5)(x+y+2)$

Treat $(x+y)$ as a single quantity, and factor as usual.

(7) $(x+y)^2+8(x+y)+15=$

(8) $(x+y)^2-(x+y)-12=$

(9) $(2x+y)^2+7a(2x+y)+10a^2=$

(10) $(x-3)^2-(x-3)-42=$

(11) $(x-5)^2+(x-5)-12=$

KUMON

J 12 a

Factorization 1

Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	1	2	3	4~

Factor the following quadratic expressions.

(1) $(x+y)^2-5(x+y)+6=$

(2) $(x+y)^2-2(x+y)-15=$

(3) $(x+y)^2+3(x+y)-18=$

(4) $(x+y)^2-7(x+y)+10=$

(5) $(x+2)^2-9(x+2)+20=$

J 12 b

(6) $(x-y)^2+4z(x-y)+3z^2=$

(7) $(x+y)^2-2z(x+y)-15z^2=$

(8) $(a+b)^2+6c(a+b)+5c^2=$

(9) $(a+b)^2-7c(a+b)+10c^2=$

(10) $(a-2b)^2-9c(a-2b)+20c^2=$

KUMON

J 13 a Factorization 1

Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	1	2	3	4~

Factor the following quadratic expressions as shown in the example.

Ex. $2x^2 + 5x - 12 = (2x - 3)(x + 4)$

$$\begin{array}{ccc} 2 & & -3 \\ & \times & \\ 1 & & 4 \end{array}$$

If you can do this step mentally, you do not need to write it out.

(1) $2x^2 + 7x + 5 =$

(2) $6x^2 + 29x - 5 =$

(3) $10x^2 - 7x - 12 =$

(4) $6x^2 - 17x + 5 =$

(5) $3x^2 - 13x + 4 =$

J 13 b

(6) $5x^2 + 16x + 3 =$

(7) $5x^2 - 2xy - 3y^2 =$

(8) $2x^2 + xy - 6y^2 =$

(9) $4x^2 - 13x - 12 =$

(10) $6x^2 - xy - 12y^2 =$

KUMON

J 14 a Factorization 1

Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	–	1	2	3~

Factor the following quadratic expressions as shown in the example.

Ex.

$$2(x+y)^2+7(x+y)+5$$
$$=[2(x+y)+5][(x+y)+1]$$
$$=(2x+2y+5)(x+y+1)$$

Treat $(x+y)$ as a single quantity, and factor as usual.

$$\begin{array}{ccc} 2 & & 5 \\ & \times & \\ 1 & & 1 \end{array}$$

If you can do this step mentally, you do not need to write it out.

(1) $2(x+y)^2+5(x+y)+3=$

(2) $2(x+y)^2+(x+y)-3=$

(3) $2(x+y)^2-9(x+y)-5=$

(4) $3(x+y)^2-13(x+y)+4=$

J 14 b

(5) $2(x+y)^2+3(x+y)+1=$

(6) $2(x+y)^2+7a(x+y)+5a^2=$

(7) $3(x-y)^2+2a(x-y)-5a^2=$

(8) $7(x+y)^2+13a(x+y)-2a^2=$

J 15 a

Factorization 1

Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	1	2	3	4~

Formulas

$$a^2 + 2ab + b^2 = (a+b)^2$$
$$a^2 - 2ab + b^2 = (a-b)^2$$

Factor the following quadratic expressions by applying the above formulas.

(1) $9x^2 - 12x + 4 =$

(2) $x^2 + 4xy + 4y^2 =$

(3) $4x^2 + 12xy + 9y^2 =$

(4) $-3ax^2 + 12axy - 12ay^2 = -3a(\qquad\qquad)$
$=$

(5) $-x^2 + 4x - 4 =$

J 15 b

Ex. $(x+y)^2+6(x+y)+9$

$=[(x+y)+3]^2$

$=(x+y+3)^2$

Treat $(x+y)$ as a single quantity, and factor by applying a formula.

(6) $(x+y)^2+4(x+y)+4=$

(7) $(x+y)^2+8(x+y)+16=$

(8) $(x+y)^2-10(x+y)+25=$

(9) $(x-y)^2-6a(x-y)+9a^2=$

(10) $(x+2y)^2-2z(x+2y)+z^2=$

KUMON

J 16 a Factorization 1

Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	1	2	3	4~

Factor the following quadratic expressions as shown in the example.

Ex.

$x^2 + 2x(a+b) + (a+b)^2$

$= [x + (a+b)]^2$

$= (x + a + b)^2$

Treat $(a + b)$ as a single quantity, and factor by applying a formula.

(1) $x^2 - 2x(a-b) + (a-b)^2 =$

(2) $x^2 + 4x(y+z) + 4(y+z)^2 =$

(3) $a^2 + 6a(b-c) + 9(b-c)^2 =$

(4) $a^2 - 8a(b+c) + 16(b+c)^2 =$

(5) $a^2 - 6a(2b-3c) + 9(2b-3c)^2 =$

J 16 b

(6) $x^2 + 6x(x + 3y) + 9(x + 3y)^2 =$

(7) $(x + 2y)^2 - 2y(x + 2y) + y^2 =$

(8) $(3x - 2y)^2 + 8y(3x - 2y) + 16y^2 =$

(9) $(x + a)^2 - 2(x + a)(y + a) + (y + a)^2 =$

(10) $(a + b)^2 + 6(a + b)(2a + b) + 9(2a + b)^2 =$

Factorization 1

Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	1	2	3	4~

Formula $a^2 - b^2 = (a + b)(a - b)$

*This formula is called
the *difference of two squares.*

Factor the following expressions by applying the above formula.

(1) $9a^2 - 4b^2 =$

(2) $5x^2 - 20 = 5(\qquad\qquad) =$

(3) $4ax^2 - 9ay^2 =$

(4) $(x + y)^2 - 4 =$

(5) $(x + y)^2 - z^2 =$

J 17 b

(6) $x^2 - (a + b)^2 =$

(7) $9 - (x - 2y)^2 =$

(8) $x^2 - (a - b)^2 =$

(9) $4a^2 - (a - b)^2 =$

(10) $(a + b + c)^2 - b^2 =$

J 18 a

Factorization 1

Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	1	2	3	4~

Factor the following expressions as shown in the examples.

Ex. $(a+b)^2-(c+d)^2$

$=[(a+b)+(c+d)][(a+b)-(c+d)]$

$=(a+b+c+d)(a+b-c-d)$

Treat $(a+b)$ and $(c+d)$ as single quantities, and factor.

(1) $(a+b)^2-(c-d)^2$

$=$

(2) $(a-b)^2-(c-d)^2$

$=$

(3) $(a-b)^2-(c+d)^2$

$=$

(4) $(x-y)^2-(a-b)^2$

$=$

(5) $(3x+2)^2-(x+3)^2$

$=$

J 18 b

Ex.

$$(3a+2b)^2 - b^2$$
$$= [(3a+2b)+b][(3a+2b)-b]$$
$$= (3a+3b)(3a+b)$$
$$= 3(a+b)(3a+b)$$

Make sure that your final answer is factored completely.

(6) $(3a+2b)^2 - (2a+3b)^2$
$=$

(7) $(2a-3b)^2 - (3a-2b)^2$
$=$

(8) $(x+2y)^2 - (2x+y)^2$
$=$

(9) $(3a-4)^2 - (2a-1)^2$
$=$

(10) $(2x-5)^2 - 9$
$=$

Factorization 1

Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	–	1	2	3~

Factor the following expressions as shown in the example. Make sure that your final answer is factored completely.

Ex.

$x^4 - y^4$

$= (x^2)^2 - (y^2)^2$

Re-write the expression in terms of x^2 and y^2, and treat them as single quantities.

$= (x^2 + y^2)(x^2 - y^2)$

$= (x^2 + y^2)(x + y)(x - y)$

You can still factor further.

(1) $x^4 - 16 =$

(2) $16x^4 - y^4 =$

(3) $x^8 - y^8 =$

(4) $x^8 - 1 =$

J 19 b

(5) $x^4 - 5x^2 - 36 = (x^2 + 4)(\qquad\qquad)$

$\qquad\qquad\qquad =$

(6) $x^4 - 8x^2 - 9 =$

(7) $x^4 - 6x^2 + 8 =$

(8) $2x^4 + x^2 - 3 =$

(9) $ax^4 - 3ax^2 - 4a =$

J 20 a

Factorization 1

Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	–	1	2	3~

Factor the following expressions as shown in the example. Make sure that your final answer is factored completely.

Ex.

$$x^4-2x^2+1$$
$$=(x^2-1)^2$$
$$=[(x+1)(x-1)]^2$$
$$=(x+1)^2(x-1)^2$$

Factor inside the parentheses.

(1) $x^4-8x^2+16=$

(2) $x^4-18x^2+81=$

(3) $x^4-2x^2y^2+y^4=$

(4) $x^5-8x^3+16x=$

(5) $x^4 - a^4 =$

(6) $x^7 - 16x^3 =$

(7) $x^4 - 8x^2y^2 + 16y^4 =$

Note Summary:

- In exercises such as
  1. $(x+y)^2 - 3(x+y) - 10 =$

     and
  2. $(a+b)^2 - (c+d)^2 =$

  Treating $(x+y)$, $(a+b)$, and $(c+d)$ as single quantities, makes the calculations easier.

- Factoring the expressions,
  1. $(x+y)^2 - 3(x+y) - 10 = (x+y-5)(x+y+2)$
  2. $(a+b)^2 - (c+d)^2 = (a+b+c+d)(a+b-c-d)$

  If you expand the terms $(x+y)^2$, $(a+b)^2$, $(c+d)^2$, the calculations become more complex.

- Helpful ways to study exercise 1.

  (i) Let $x+y=A$

  Substituting this value into
  $(x+y)^2 - 3(x+y) - 10$,
  results to $A^2 - 3A - 10$.

  (ii) $(\underline{x+y})^2 - 3(\underline{x+y}) - 10$

  $((x+y))^2 - 3((x+y)) - 10$

  You can either underline or circle the terms that are to be treated as single quantities.

KUMON

J 21 a

Factorization 2

Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	1	2	3	4~

Factor the following expressions as shown in the examples.

Ex. $x(a-2b)+y(a-2b)=(a-2b)(x+y)$

$2a(x+y)+4(x+y)=2(x+y)(a+2) \leftarrow$ Don't forget to factor out the 2 as it is also a common factor.

(1) $x(2a-b)-y(2a-b)=$

(2) $3x(a+1)+6(a+1)=$

(3) $4a(x+3)+6(x+3)=$

(4) $3a(x-3)+3(x-3)=$

(5) $3x(a-2)-6(a-2)=$

J 21 b

Ex.

$$a(x-3)+b(\mathbf{3-x})$$
$$=a(x-3)-b(\mathbf{x-3})$$
$$=(x-3)(a-b)$$

Since $(3-x)=-(x-3)$, re-write the expression so that $(x-3)$ becomes the common factor.

(6) $2(x-2)+a(2-x)$
$=$

(7) $x(a-b)-2(b-a)$
$=$

(8) $5x(2a-b)+2y(b-2a)$
$=$

(9) $3ab(2x-y)-5a^2(y-2x)$
$=$

(10) $2b(x-2y)-4a(2y-x)$
$=$

J 22 a

Factorization 2

Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	1	2	3	4~

Factor the following expressions as shown in the examples. Make sure that your final answer is factored completely.

Ex.

$$x^2(a-2)+y^2(2-a)$$
$$=x^2(a-2)-y^2(a-2)$$
$$=(a-2)(x^2-y^2)$$
$$=(a-2)(x+y)(x-y)$$

$(2-a)=-(a-2)$

Factor the term (x^2-y^2).

(1) $9x^2(a-b)+4(b-a)$
$=$

(2) $x^2(a-b)+16(b-a)$
$=$

(3) $4x^2(x-2y)+9y^2(2y-x)$
$=$

(4) $a^2(x-2y)+4b^2(2y-x)$
$=$

(5) $-a^2(3y-x)-9(x-3y)$
$=$

J 22 b

Ex.

$$x^2(a-b)+x(a-b)+2(b-a)$$
$$=x^2(a-b)+x(a-b)-2(a-b)$$
$$=(a-b)(x^2+x-2)$$
$$=(a-b)(x+2)(x-1)$$

$(b-a)=-(a-b)$

(6) $x^2(a-b)+x(b-a)+2(b-a)$
$=$

(7) $x^2(a-b)-6x(a-b)-9(b-a)$
$=$

(8) $2x^2(a-b)+x(b-a)+6(b-a)$
$=$

(9) $(a-b)(x^2-5)+(b-a)(3x+5)$
$=$

(10) $(a-b)(2x^2+9)-(b-a)(7x-6)$
$=$

KUMON®

J 23 a

Factorization 2

Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	–	1	2	3~

Factor the following expressions as shown in the example.

Ex.

$$2(a-b)\underset{\sim}{+}(b-a)^2$$
$$=2(a-b)\underset{\sim}{+}(a-b)^2$$
$$=(a-b)[2+(a-b)]$$
$$=(a-b)(2+a-b)$$

$(b-a)^2=(a-b)^2$

Because the quantity is squared, when we factor out a -1, the sign of the term does not change.

(1) $3(a-b)+(b-a)^2$
$=$

(2) $3(a-b)-(b-a)^2$
$=$

(3) $(y-x)^2+3(x-y)$
$=$

Note that, when we expand the following expressions, we get the same result for both.
$(b-a)^2=b^2-2ab+a^2$, $(a-b)^2=a^2-2ab+b^2$
Thus, factoring out a -1 from these expressions does not affect the overall sign of the quantity.

J 23 b

(4) $2(2y-x)^2+4a(x-2y)$

$=$

(5) $a(2y-x)^2+2a^2(x-2y)$

$=$

(6) $x(3b-2a)^2-2x^2(2a-3b)$

$=$

(7) $2(x-y)-(y-x)^2$

$=$

(8) $2a^2(x-3y)+a(3y-x)^2$

$=$

J 24 a   Factorization 2

Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	–	1	2	3~

Factor the following expressions as shown in the example.

Ex.
$xy(x-y)+3y(y-x)^2$
$=xy(x-y)+3y(x-y)^2$
$=y(x-y)[x+3(x-y)]$
$=y(x-y)(4x-3y)$

$(y-x)^2=(x-y)^2$

(1) $2x(x-y)-3(y-x)^2$
$=$

(2) $x(2y-x)^2+2x^2(x-2y)$
$=$

(3) $4x^2(x-3y)-2x(3y-x)^2$
$=$

(4) $xy(x-2y)+3y(2y-x)^2$
$=$

J 24 b

(5) $2a^2(a-3b)+a(3b-a)^2$
$=$

(6) $(a-b)^2(3x-5y)-(b-a)^2(x-y)$
$=$

(7) $3x^2y(x-y)-6xy^2(y-x)^2$
$=$

(8) $5xy^2(2y-3x)-15x^2(3x-2y)^2$
$=$

KUMON®

J 25 a — Factorization 2

Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	–	1	2	3~

Factor the following expressions as shown in the example.

Ex.
$$2(x-y)^2+(y-x)^3$$
$$=2(x-y)^2-(x-y)^3$$
$$=(x-y)^2[2-(x-y)]$$
$$=(x-y)^2(2-x+y)$$

$(y-x)^3=-(x-y)^3$

Because the quantity is cubed, when we factor out a -1, the sign of the term changes.

(1) $(x-y)^2+3(y-x)^3$
$=$

(2) $x(a-b)^2+2y(b-a)^3$
$=$

(3) $4a^2(x-3)^2-2(3-x)^3$
$=$

(4) $x^2y(x-3y)^2+xy^2(3y-x)^3$
$=$

J 25 b

(5) $6xy^2(3y - x) - 3x^2(x - 3y)^2$
$=$

(6) $5x^2y(x - y)^3 - 10xy^3(y - x)^2$
$=$

(7) $2x^2(x - 2y)^2 + 6xy^2(2y - x)$
$=$

<u>Note Summary:</u>

A. To factor expressions, sometimes we need to factor out a -1 from a term, so that it can be the same as another term in the expression, allowing us to factor that term out. As we factor out a -1, the terms inside the parentheses change signs. We have two cases for the sign outside of the parentheses:

1. When the power outside of the parentheses is an even number, the sign of the overall term remains the same:
$(b - a)^2 = (a - b)^2$, $(b - a)^4 = (a - b)^4$
2. When the power outside of the parentheses is an odd number, the sign of the overall term changes.
$(b - a) = -(a - b)$, $(b - a)^3 = -(a - b)^3$

B. The following example shows what happens when we alter an expression in two different ways. Either way is correct.

1. $a(x - 3y)^2 - b(3y - x)^3 = a(x - 3y)^2 + b\boldsymbol{(x - 3y)^3}$
2. $a(x - 3y)^2 - b(3y - x)^3 = a\boldsymbol{(3y - x)^2} - b(3y - x)^3$

Factorization 2

Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	1	2	3	4~

Factor the following expressions as shown in the example.

Ex.

$$ax + ay + bx + by$$
$$= a(x + y) + b(x + y)$$
$$= (x + y)(a + b)$$

Group terms that have common factors.
ax and ay have a in common
bx and by have b in common

(1) $ax - ay + bx - by$
$=$

(2) $ax + ay - bx - by$
$=$

(3) $ax - ay - bx + by$
$=$

(4) $ab + ac + bd + cd$
$=$

(5) $ab - ac + bd - cd$
$=$

Note: In the above example, we get the same result if we group the expression into x and y terms.

$$ax + ay + bx + by = x(a + b) + y(a + b) = (a + b)(x + y)$$

J 26 b

(6) $ab + cd - ac - bd$
$=$

(7) $ab + cd + bc + ad$
$=$

(8) $ab + cd - bd - ac$
$=$

(9) $ab - cd + bd - ac$
$=$

(10) $ab - cd - bd + ac$
$=$

J 27 a Factorization 2

Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	1	2	3	4~

Factor the following expressions.

(1) $a^2 + xy + ax + ay$
$=$

(2) $a^2 - xy + ax - ay$
$=$

(3) $a^2 - xy - ax + ay$
$=$

(4) $2ax + 3by + 3bx + 2ay$
$=$

(5) $2ax - 3by + 3bx - 2ay$
$=$

J 27 b

(6) $a^2 + 2ax + ab + 2bx$
$=$

(7) $a^2 + 2ax - ab - 2bx$
$=$

(8) $2ax - ab - a^2 + 2bx$
$=$

(9) $3ax + 2bx + 3ay + 2by$
$=$

(10) $2ax - 3by - 6bx + ay$
$=$

Factorization 2

Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	1	2	3	4~

Factor the following expressions as shown in the example.

Ex. $x^3 + 4x^2 + 2x + 8$

$= x^2(x+4) + 2(x+4)$

$= (x+4)(x^2+2)$

Factor by grouping:
x^3 and $4x^2$ have x^2 in common.
$2x$ and 8 have 2 in common.

(1) $x^3 - 4x^2 + 2x - 8$
$=$

(2) $x^3 + 4x^2 - 2x - 8$
$=$

(3) $x^3 - 2x - 4x^2 + 8$
$=$

(4) $x^3 + x^2 + x + 1$
$=$

(5) $x^3 - x^2 + x - 1$
$=$

J 28 b

(6) $x^3 - 2x^2 + x - 2$
$=$

(7) $x^3 + x^2 - 4x - 4$
$=$

(8) $x^3 - x - x^2 + 1$
$=$

(9) $x^2y^2 - x^2 + y^2 - 1$
$=$

(10) $x^2y^2 - x^2 - y^2 + 1$
$=$

Factorization 2

Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	–	1	2	3~

Factor the following expressions.

(1) $x^2 - xy - x + y$
$=$

(2) $x^3 + x^2y - xy^2 - y^3$
$=$

(3) $xy + 1 + x + y$
$=$

(4) $x^2y + y^2z + x^2z + y^3$
$=$

(5) $1 - x - x^2 + x^3$
$=$

J 29 b

Ex.

$$a^2 + 2ab + b^2 + ax + bx$$
$$= (a+b)^2 + x(a+b)$$
$$= (a+b)[(a+b) + x]$$
$$= (a+b)(a+b+x)$$

$$a^2 + 2ab + b^2 = (a+b)^2$$

(6) $a^2 - 2ab + b^2 - ax + bx$
$=$

(7) $ab - ac - b^2 + 2bc - c^2$
$=$

(8) $a^2 - b^2 - ac + bc$
$=$

(9) $ab + ac - b^2 - 2bc - c^2$
$=$

J 30 a Factorization 2

Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	–	1	2	3~

Factor the following expressions. Make sure that your final answer is factored completely.

(1) $x(2y-x)^2+2x^2(x-2y)$
$=$

(2) $xy(x-2y)+3y(2y-x)^2$
$=$

(3) $(a-2b)(3x-5y)+(2b-a)(x-y)$
$=$

(4) $4xy^2(3y-x)-2x^2(x-3y)^2$
$=$

(5) $5xy^2(2y-3x)-15x^2(3x-2y)^2$
$=$

J 30 b

(6) $2xy - 4z^2 - 2xz + 4yz$
$=$

(7) $6 - 9x^2 + 12y - 18x^2y$
$=$

(8) $ax^2y^2 - ax^2 - ay^2 + a$
$=$

Note Summary:

A. To factor an expression such as $ax + ay + bx + by$, group the terms that have common factors.
a can be factored out from the terms, $ax + ay = a(x + y)$
b can be factored out from the terms, $bx + by = b(x + y)$
Each resulting group has a factor of $(x + y)$.

B. There are two different ways to factor the expression $ax + ay + bx + by$ by grouping.

1. Factoring out a and b:
$ax + ay + bx + by = a(x + y) + b(x + y) = (x + y)(a + b)$
2. Factoring out x and y:
$ax + ay + bx + by = x(a + b) + y(a + b) = (a + b)(x + y)$

KUMON®

J 31 a

Factorization 3

Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	–	1	2	3~

Factor the following expressions as shown in the examples.

Ex. $x^2+(2a+b)x+2ab$

$=(x+2a)(x+b)$

Look for two terms that when multiplied together will give $2ab$ and, when added together will give $2a+b$. The two terms that satisfy these conditions are $2a$ and b.

(1) $x^2+(a+3b)x+3ab=$

(2) $x^2-(2a+b)x+2ab=$

(3) $x^2+(2a-b)x-2ab=$

(4) $x^2-(2a-b)x-2ab=$

(5) $x^2-(a-3b)x-3ab=$

J 31 b

Ex.

$$x^2 + 2x - a(a+2)$$

$$= (x-a)[x + (a+2)]$$

$$= (x-a)(x+a+2)$$

Determine which factor, a or $(a+2)$, will take the negative sign, $[\,-\,]$, so that when we add the two terms, we get 2.

Note: $\begin{bmatrix} a-(a+2)=-2 \\ -a+(a+2)=2 \end{bmatrix}$

(6) $x^2 + 3x - a(a+3)$
$=$

(7) $x^2 + bx - a(a+b)$
$=$

(8) $x^2 - bx - a(a+b)$
$=$

(9) $x^2 + 2x - a(a-2)$
$=$

J 32 a

Factorization 3

Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	–	1	2	3~

Factor the following expressions as shown in the examples.

Ex.

$$x^2 + (2y+5)x + (y+6)(y-1)$$

$$= [x + (y+6)][x + (y-1)]$$

$$= (x+y+6)(x+y-1)$$

Find two terms, with the appropriate signs, that when multiplied together will give $(y+6)(y-1)$, and when added together will give $2y+5$.

(1) $x^2 + (3y+4)x + (2y+3)(y+1)$
$=$

(2) $x^2 + (3y+5)x + (2y+3)(y+2)$
$=$

(3) $x^2 - (2y+5)x + (y+6)(y-1)$
$=$

(4) $x^2 - (3y+4)x + (2y+3)(y+1)$
$=$

J 32 b

Ex.

$$x^2 + (y+4)x - (2y+1)(3y+5)$$

$$= [x - (2y+1)][x + (3y+5)]$$

$$= (x - 2y - 1)(x + 3y + 5)$$

Determine which factor, $(2y+1)$ or $(3y+5)$, will take the negative sign, $[-]$, so that when we add the two terms, we get $y+4$.

Note: $\begin{bmatrix} (2y+1) - (3y+5) = -y-4 \\ -(2y+1) + (3y+5) = y+4 \end{bmatrix}$

(5) $x^2 + (y-7)x - (y+6)(2y-1)$
$=$

(6) $x^2 - (y+2)x - (2y+3)(y+1)$
$=$

(7) $x^2 + (y+1)x - (2y+3)(y+2)$
$=$

(8) $x^2 - (y-4)x - (2y-1)(3y-5)$
$=$

J 33 a

Factorization 3

Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	–	1	2	3~

Factor the following expressions as shown in the example.

Ex.

$$x^2+(3y+4)x+(2y^2+5y+3)$$
$$=x^2+(3y+4)x+(y+1)(2y+3)$$
$$=[x+(y+1)][x+(2y+3)]$$
$$=(x+y+1)(x+2y+3)$$

First factor the quadratic $(2y^2+5y+3)$.

(1) $x^2+(3y+5)x+(2y^2+7y+6)$
$=$

(2) $x^2-(3y+4)x+(2y^2+3y-5)$
$=$

(3) $x^2-(5y-6)x+(6y^2-13y+5)$
$=$

J 33 b

(4) $x^2 + yx - (6y^2 - 5y + 1)$
$=$

(5) $x^2 - (y + 4)x - (2y^2 + y - 3)$
$=$

(6) $x^2 - (y - 1)x - (2y^2 + 11y + 12)$
$=$

(7) $x^2 - (2y + 1)x - (3y^2 - 11y + 6)$
$=$

(8) $x^2 - yx - (6y^2 - 5y + 1)$
$=$

J 34 a

Factorization 3

Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	–	1	2	3~

Factor the following expressions as shown in the example.

Ex.

$x^2+5xy+6x+6y^2+13y+5$

$=x^2+(5y+6)x+(6y^2+13y+5)$

$=x^2+(5y+6)x+(2y+1)(3y+5)$

$=[x+(2y+1)][x+(3y+5)]$

$=(x+2y+1)(x+3y+5)$

Arranging the expression in standard polynomial form, with x as the variable, first collect and group together all of the x^2, x, and "non-x" terms.

Factor the quadratic that is formed by all the "non-x" terms, the constants.

(1) $x^2+3xy+2y^2+5x+7y+6$
$=$

(2) $x^2-3xy+2y^2-5x+7y+6$
$=$

(3) $x^2+3xy+2y^2-5x-7y+6$
$=$

J 34 b

(4) $x^2 - 5xy + 6y^2 + 6x - 13y + 5$
$=$

(5) $x^2 + 2y^2 + 3 + 3xy + 4x + 5y$
$=$

(6) $x^2 - xy - 2y^2 - 4x + 11y - 5$
$=$

(7) $x^2 + xy - 2y^2 - x - 11y - 12$
$=$

J 35 a

Factorization 3

Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	–	1	2	3~

Factor the following expressions.

(1) $x^2+3xy+2y^2+4x+7y+3$
$=$

(2) $x^2-5xy+6y^2-3x+7y+2$
$=$

(3) $x^2-3y^2+2xy+4x-8y-5$
$=$

(4) $x^2+6x+5-2y^2-xy-9y$
$=$

J 35 b

(5) $x^2 - 4y + 3y^2 + 4xy - 4$
$=$

(6) $x^2 - xy - 6y^2 - x + 13y - 6$
$=$

(7) $x^2 + 3y^2 - 4xy + 4x - 16y - 12$
$=$

(8) $x^2 - y - 6y^2 + xy - 7x + 12$
$=$

J 36 a

Factorization 3

Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	–	1	2	3~

Factor the following expressions as shown in the examples.

Ex.

$$x^2+4xy+4x+4y^2+8y+4$$
$$=x^2+4x(y+1)+4(y^2+2y+1)$$
$$=x^2+4x(y+1)+4(y+1)^2$$
$$=[x+2(y+1)]^2$$
$$=(x+2y+2)^2$$

Apply the formula
$a^2+2ab+b^2=(a+b)^2$

(1) $x^2+2xy+2x+y^2+2y+1$
$=$

(2) $x^2-2xy-2x+y^2+2y+1$
$=$

(3) $x^2-4xy-4x+4y^2+8y+4$
$=$

(4) $x^2-6xy-6x+9y^2+18y+9$
$=$

J 36 b

Ex.

$$x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$$
$$= x^2 + 2x(y+z) + (y^2 + 2yz + z^2)$$
$$= x^2 + 2x(y+z) + (y+z)^2$$
$$= [x + (y+z)]^2$$
$$= (x+y+z)^2$$

Arrange the expression in standard polynomial form, with x as the variable.

(5) $x^2 + y^2 + 9z^2 + 2xy + 6xz + 6yz$

$=$

(6) $x^2 + 4y^2 + z^2 + 4xy + 2xz + 4yz$

$=$

(7) $x^2 + 4y^2 + 9z^2 + 4xy + 6xz + 12yz$

$=$

J 37 a

Factorization 3

Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	–	1	–	2~

Factor the following expressions as shown in the examples.

Ex.

$$x^2+3xy+5x+2y^2+8y+6$$
$$=x^2+(3y+5)x+2(y^2+4y+3)$$
$$=x^2+(3y+5)x+2(y+3)(y+1)$$
$$=[x+2(y+1)][x+(y+3)]$$
$$=(x+2y+2)(x+y+3)$$

The two terms that result to $2(y+3)(y+1)$ when multiplied, and $(3y+5)$ when added, are $2(y+1)$ and $(y+3)$.

(1) $x^2+3xy-3x+2y^2-8y-10$
$=$

(2) $x^2+3xy-3x+2y^2-2y-4$
$=$

(3) $x^2-xy-x-2y^2+14y-20$
$=$

J 37 b

Ex.

$x^2 + 4xy + 4x + 3y^2 + 6y + 3$

$= x^2 + 4x(y+1) + 3(y^2 + 2y + 1)$

$= x^2 + 4x(y+1) + 3(y+1)^2$

$= [x + 3(y+1)][x + (y+1)]$

$= (x + 3y + 3)(x + y + 1)$

The two terms that result to $3(y+1)^2$ when multiplied and $4(y+1)$ when added, are $3(y+1)$ and $(y+1)$.

(4) $x^2 + 5xy + 5x + 6y^2 + 12y + 6$
$=$

(5) $x^2 + 5xy + 5x - 6y^2 - 12y - 6$
$=$

(6) $x^2 + 4xy + 4x - 12y^2 - 24y - 12$
$=$

KUMON

J 38 a

Factorization 3

Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	–	1	2	3~

Factor by using the "cross-multiplication" method, as shown in the example.

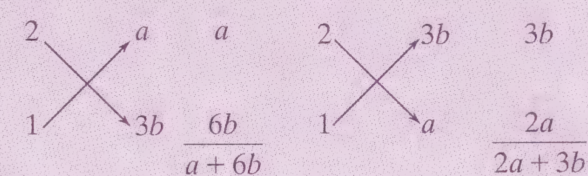
Ex. $2x^2 + (a+6b)x + 3ab$

$= (2x+a)(x+3b)$

The expression will factor into $(2x + \triangle)(x + \square)$. Determine what the two missing terms are by cross multiplication.

There are two possibilities for the set-up of the "cross-multiplication" diagram:

A. B.



(1) $2x^2 - (a+6b)x + 3ab$

$=$

(2) $2x^2 + (a-6b)x - 3ab$

$=$

(3) $2x^2 + (2a+3b)x + 3ab$

$=$

J 38 b

(4) $2x^2 + (2a - 3b)x - 3ab$
$=$

(5) $2x^2 - (2a - 3b)x - 3ab$
$=$

(6) $2x^2 + (3a + 8)x + a(a + 4)$
$=$

(7) $2x^2 + (a + 6)x - a(a + 3)$
$=$

(8) $2x^2 + (3a + 4b)x + a(a + 4b)$
$=$

J 39 a

Factorization 3

Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	–	1	2	3~

Factor each of the following expressions as shown in the example.

Ex.

$$
\begin{aligned}
&2x^2 + 5xy + 3y^2 + 6x + 7y + 4\\
&= 2x^2 + (5y + 6)x + (3y^2 + 7y + 4)\\
&= 2x^2 + (5y + 6)x + (3y + 4)(y + 1)\\
&= [2x + (3y + 4)][x + (y + 1)]\\
&= (2x + 3y + 4)(x + y + 1)
\end{aligned}
$$

Since
$2 \times (y + 1) + 1 \times (3y + 4)$
$= 5y + 6$

(1) $2x^2 + 7xy + 3y^2 + 9x + 7y + 4$
$=$

(2) $2x^2 + 7xy + 3y^2 - 9x - 7y + 4$
$=$

(3) $2x^2 + 8xy + 6y^2 + 11x + 13y + 5$
$=$

J 39 b

(4) $3x^2+7xy+2y^2+11x+7y+6$
$=$

(5) $2x^2-7xy+3y^2-9x+7y+4$
$=$

(6) $2x^2-8xy+6y^2-11x+13y+5$
$=$

(7) $3x^2-7xy+2y^2-11x+7y+6$
$=$

J 40 a Factorization 3

Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	–	1	2	3~

Factor the following expressions.

(1) $x^2 + 3(y+z)x + (y+2z)(2y+z)$
$=$

(2) $x^2 + (y-z)x - (y+2z)(2y+z)$
$=$

(3) $x^2 - (y-z)x - (y+2z)(2y+z)$
$=$

(4) $a^2 - 2b^2 - 3c^2 - ab - 2ac - 5bc$
$=$

J 40 b

(5) $a^2 - 8b^2 + 2c^2 + 2ab + 3ac$
$=$

(6) $x^2 - 4y^2 + 3x - 2y + 2$
$=$

(7) $2x^2 - (5a - 4b)x - (a + 2b)(3a - b)$
$=$

(8) $2x^2 - 7xy + 3y^2 + 9x - 7y + 4$
$=$

J 41 a

Factorization 4

Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	–	1	2	3~

Factor the following expressions.

(1) $2x^2 - 3xy - 2y^2 - 2x - 11y - 12$
$=$

(2) $2x^2 + 3xy - 2y^2 + 2x - 11y - 12$
$=$

(3) $2x^2 + 7xy + 3y^2 + 13x + 14y + 15$
$=$

J 41 b

(4) $2x^2 - 7xy + 6y^2 + 7x - 11y + 3$
$=$

(5) $2x^2 + xy - y^2 + 3x - 3y - 2$
$=$

(6) $2x^2 + xy - y^2 + 3x + 1$
$=$

(7) $2x^2 - 5xy - 3y^2 - 14y - 8$
$=$

J 42 a

Factorization 4

Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	–	1	–	2~

Factor the following expressions as shown in the example.

Ex.

$$2a^2 + 2b^2 + c^2 + 4ab + 3ac + 3bc$$
$$= 2a^2 + (4b + 3c)a + (2b^2 + 3bc + c^2)$$
$$= 2a^2 + (4b + 3c)a + (2b + c)(b + c)$$
$$= [2a + (2b + c)][a + (b + c)]$$
$$= (2a + 2b + c)(a + b + c)$$

(1) $2a^2 + b^2 + 2c^2 + 3ab + 5ac + 3bc$
$=$

(2) $2a^2 + 2b^2 + 2c^2 - 4ab - 5ac + 5bc$
$=$

J 42 b

(3) $2a^2-3b^2-4c^2+5ab-2ac+8bc$
$=$

(4) $3a^2-6b^2-2c^2-7ab+5ac+7bc$
$=$

(5) $2a^2-2b^2-3c^2+3ab+5ac-5bc$
$=$

(6) $2a^2-2b^2-c^2-3ab+ac+3bc$
$=$

J 43 a

Factorization 4

Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	–	–	1	2~

1. Arrange the expressions, in standard polynomial form, with b as the variable. Then factor the polynomials as shown in the example.

Ex.

$$2a^2 + 2b^2 + c^2 + 4ab + 3ac + 3bc$$
$$= 2b^2 + (4a + 3c)b + (2a^2 + 3ac + c^2)$$
$$= 2b^2 + (4a + 3c)b + (2a + c)(a + c)$$
$$= [2b + (2a + c)][b + (a + c)]$$
$$= (2b + 2a + c)(b + a + c)$$

(1) $3a^2 + 2b^2 + 6c^2 + 7ab + 11ac + 7bc$
$=$

(2) $2a^2 + 2b^2 + 2c^2 - 4ab - 5ac + 5bc$
$=$

* Looking at the example on J42a and the example above, we can see that we get the same result by arranging the expression with a, b, or c as the variable.

J 43 b

2. $3a^2 - 5ab + 2b^2 - a - b - 10$

Factor the expression. Begin by arranging the expression as follows:

In exercise (1), arrange it in standard polynomial form, with a as the variable.
In exercise (2), arrange it in standard polynomial form, with b as the variable.

(1) With a as the variable,

$$3a^2 - 5ab + 2b^2 - a - b - 10$$
$$=$$

(2) With b as the variable,

$$3a^2 - 5ab + 2b^2 - a - b - 10$$
$$=$$

J 44 a

Factorization 4

Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	–	1	–	2~

1. Factor the following expressions by using the difference of two squares. (i.e. by forming the given expression into two terms as $A^2 - B^2$)

Ex.

$$4a^2 + 4ab + b^2 - c^2$$
$$= (4a^2 + 4ab + b^2) - c^2$$
$$= (2a + b)^2 - c^2$$
$$= [(2a + b) + c][(2a + b) - c]$$
$$= (2a + b + c)(2a + b - c)$$

(1) $a^2 + 6ab + 9b^2 - c^2$
$=$

(2) $4a^2 - 12ab + 9b^2 - 4c^2$
$=$

(3) $4a^2 - b^2 + 6bc - 9c^2$
$=$

J 44 b

2. $a^2 + 2ab + b^2 - x^2 - 6x - 9$

Factor the expression. Begin by arranging the expression as follows:

In exercise (1), arrange it in standard polynomial form, with *a* as the variable.
In exercise (2), arrange it in standard polynomial form, with *b* as the variable.
In exercise (3), arrange it as the difference of two squares.

(1) With *a* as the variable,

$$a^2 + 2ab + b^2 - x^2 - 6x - 9$$
$$=$$

(2) With *b* as the variable,

$$a^2 + 2ab + b^2 - x^2 - 6x - 9$$
$$=$$

(3) As the difference of two squares,

$$a^2 + 2ab + b^2 - x^2 - 6x - 9$$
$$=$$

J 45 a Factorization 4

Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	–	1	2	3~

Factor the following expressions. You may use any method.

(1) $x^2 - y^2 - 4x + 4$
$=$

(2) $x^2 - y^2 - 6y - 9$
$=$

(3) $4x^2 - 12xy + 9y^2 - 9$
$=$

J 45 b

(4) $x^2 - y^2 + 4ay - 4a^2$

$=$

(5) $x^2 - 6ax - y^2 + 9a^2$

$=$

(6) $9x^2 + 4yz - 4y^2 - z^2$

$=$

(7) $x^2 + 4y^2 - z^2 - 4 + 4xy - 4z$

$=$

J 46 a

Factorization 4

Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	–	1	2	3~

Factor the following expressions. You may use any method.

(1) $a^2 - b^2 - c^2 + d^2 + 2ad + 2bc$
$=$

(2) $2ad - 2bc - a^2 + b^2 + c^2 - d^2$
$=$

(3) $a^2 + b^2 + 2bc - 2ca - 2ab$
$=$

J 46 b

(4) $ax^2 - bx^2 - 2bx + 2ax - 3a + 3b$
$=$

(5) $2x^2 + xy - y^2 + 3x - 3y - 2$
$=$

(6) $2x^2 + xy - y^2 + 3x + 1$
$=$

(7) $x^2 - 4y^2 - x + 6y - 2$
$=$

KUMON

J 47 a

Factorization 4

Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	1	2	3	4~

Formulas

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Factor each expression by applying the above formulas.

(1) $x^3 + 8y^3 =$

(2) $8x^3 - 27y^3 =$

(3) $a^3 - 8b^3 =$

(4) $8a^3 + 27b^3 =$

(5) $a^3 - 64 =$

J 47 b

(6) $27a^3 - 8b^3 =$

(7) $1 - x^3 =$

(8) $a^3 + 8b^6 =$

(9) $a^3 - 27b^9 =$

(10) $x^6 + y^9 =$

KUMON®

J 48 a

Factorization 4

Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	–	1	2	3~

Formulas

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Factor each expression by applying the above formulas.

(1) $125a^3 + 8b^3 =$

(2) $27x^3 - 64 =$

(3) $2x^3 + 16 = 2(\qquad\qquad) =$

(4) $64a^4 - 27a =$

(5) $64x - x^4 =$

J 48 b

(6) $(a+b)^3-8b^3$
$=$

(7) $8(x+y)^3-y^3$
$=$

(8) $(a+b)^3-b^3$
$=$

(9) $(a+b)^3-(b-c)^3$
$=$

KUMON

J 49 a

Factorization 4

Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	–	1	2	3~

Factor the following expressions.

(1) $a^6 - b^6$

$= (a^2 - b^2)(a^4 + a^2b^2 + b^4)$

$= (a + \boxed{})(a - \boxed{})[(a^2 + b^2)^2 - a^2b^2]$

$= (\qquad)(\qquad)[(a^2 + b^2) + ab][(\qquad) - ab]$

$=$

Note

$a^6 - b^6 = [(a^2)^3 - (b^2)^3]$

(2) $a^6 - b^6$

$= (a^3 + \boxed{})(a^3 - \boxed{})$

$=$

Note

$a^6 - b^6 = [(a^3)^2 - (b^3)^2]$

(3) $a^6 + b^6$

$=$

(4) $a^9 - b^9$

$=$

(5) $a^{12} + b^{12}$

$=$

* Note that in exercises (1) and (2) both methods shown give the same answer.

J 49 b

Ex.

$$(x^3 + y^3) + xy(x + y)$$
$$= (x + y)(x^2 - xy + y^2) + xy(x + y)$$
$$= (x + y)(x^2 + y^2)$$

(6) $xy(x - y) + x^3 - y^3$

$=$

(7) $a^3 + b^3 + a + b$

$=$

(8) $a^3 - b^3 - 3ab(a - b)$

$=$

(9) $4 - x^2 + 4x^3 - x^5$

$=$

J 50 a

Factorization 4

Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	–	1	2	3~

Factor the following expressions.

(1) $2x^2 - 7xy + 3y^2 + 9x - 7y + 4$
$=$

(2) $3x^2 + 7xy + 2y^2 + 11x + 7y + 6$
$=$

(3) $2x^2 - 2y^2 - z^2 + 3yz + zx - 3xy$
$=$

(4) $2a^2 + 6b^2 - 18c^2 + 3bc - 7ab$
$=$

J 50 b

(5) $a^2 + 5ab + 5a - 6b^2 - 12b - 6$
$=$

(6) $a^2 - 4b^2 - 3c^2 + 8bc - 2ac$
$=$

(7) $a^2 - 4b^2 + 4bc - c^2$
$=$

(8) $1 - x^2 + x^3 - x^5$
$=$

KUMON

J 51 a

Factorization 5

Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	–	1	2	3~

Factor the following expressions as shown in the example.

Ex.

$x^4 \underline{- 6x^2} + 1$ — Take $-6x^2$ and break it up into two terms, $-2x^2$ and $-4x^2$.

$= x^4 \underline{- 2x^2} + 1 \underline{- 4x^2}$ — Factor $x^4 - 2x^2 + 1$ and rewrite the term $-4x^2$ as $-(2x)^2$.

$= (x^2 - 1)^2 - (2x)^2$ — Factor the *difference of two squares.*

$= (x^2 - 1 + 2x)(x^2 - 1 - 2x)$

$= (x^2 + 2x - 1)(x^2 - 2x - 1)$

(1) $x^4 - 11x^2 + 1 =$

(2) $x^4 - 27x^2 + 1 =$

(3) $x^4 - 7x^2 + 1 = (x^4 + 2x^2 + 1) - \boxed{}$

$=$

J 51 b

(4) $x^4 - 23x^2 + 1 =$

(5) $a^4 - 13a^2 + 4 =$

(6) $x^4 + 2x^2 + 9 =$

(7) $x^4 + x^2 + 1 =$

(8) $a^4 + a^2b^2 + b^4 =$

KUMON

J 52 a

Factorization 5

Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	–	1	2	3~

Factor the following expressions.

(1) $x^4 - 18x^2 + 1 =$

(2) $x^4 - 14x^2 + 1 =$

(3) $x^4 - 38x^2 + 1 =$

(4) $x^4 - 34x^2 + 1 =$

J 52 b

(5) $4x^4 + 3x^2 + 1 =$

(6) $4x^4 + 11x^2 + 9 =$

(7)* $x^4 - 10x^2 + 9$
$=$

*The answer to this exercise is the product of four factors.

(8) $9x^4 - 13x^2 + 4 =$

Factorization 5

Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	–	1	–	2~

Factor the following expressions as shown in the examples.

Ex.

$(x^2+x-6)(x^2+x-2)+3$

$=(A-6)(A-2)+3$

$=A^2-8A+15$

$=(A-3)(A-5)$

$=(x^2+x-3)(x^2+x-5)$

Let $x^2+x=A$, and rewrite the expression in terms of A.

Substitute A with its value, x^2+x.

(1) $(x^2+x-7)(x^2+x-5)-8$

$=$

(2) $(x^2+2x-8)(x^2+2x+1)-10$

$=$

(3) $(x^2+5x)(x^2+5x+6)-16$

$=$

J 53 b

Ex.

$$(x^2 + x - 5)(x^2 + 2x - 5) - 12x^2$$

Let $x^2 - 5 = A$

$$= (A + x)(A + 2x) - 12x^2$$

$$= A^2 + 3xA - 10x^2$$

$$= (A - 2x)(A + 5x)$$

$$= (x^2 - 2x - 5)(x^2 + 5x - 5)$$

(4) $(x^2 - 2x - 7)(x^2 + 3x - 7) - 6x^2$

$=$

(5) $(x^2 + 3x + 3)(x^2 - 4x + 3) + 6x^2$

$=$

(6)* $(x^2 - 7x - 18)(x^2 + 3x - 18) + 24x^2$

$=$

Factorization 5

Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	–	1	–	2~

Factor the following expressions as shown in the example.

Ex.

$$(x+1)(x+2)(x+3)(x+4)-8$$

Multiply $(x+2)$ by $(x+3)$ and then $(x+1)$ by $(x+4)$

$$=(x^2+5x+6)(x^2+5x+4)-8$$

Treat x^2+5x as a single quantity, as if we let $x^2+5x=A$.

$$=(x^2+5x)^2+10(x^2+5x)+16$$

$$=(x^2+5x+2)(x^2+5x+8)$$

(1) $(x-1)(x-2)(x-3)(x-4)-15$

$=$

(2) $(x+1)(x+2)(x+3)(x+4)-3$

$=$

(3) $(x-1)(x-3)(x+2)(x+4)+24$
$=$

(4) $x(x+1)(x+2)(x+3)-15$
$=$

(5) $(x+1)(x+2)(x+3)(x+4)+1$
$=$

KUMON

J 55 a

Factorization 5

Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	–	–	1	2~

Factor the following expressions as shown in the example.

Ex. $x^2(b-c)+\underline{b^2(c-x)}+\underline{c^2(x-b)}$

Arrange the terms in standard polynomial form, with x as the variable. The first term already shows the x^2 terms. Leave the first term as is, and expand the underlined terms.

$=(b-c)x^2+b^2c-b^2x+c^2x-c^2b$

Collect and group together all the x and "non-x" terms; the constants.

$=(b-c)x^2-(b^2-c^2)x+bc(b-c)$

$=(b-c)x^2-(b+c)(b-c)x+bc(b-c)$

Factor out the common term $(b-c)$.

$=(b-c)[x^2-(b+c)x+bc]$

$=(b-c)(x-b)(x-c)$

(1) $a^2(b-c)+b^2(c-a)+c^2(a-b)$
$=$

(2) $x^2(y+z)+y^2(z+x)+z^2(x+y)+2xyz$
$=$

J 55 b

(3) $bc(b-c) - ca(c+a) + ab(a+b)$
$=$

(4) $a^2(b+c) + b^2(c-a) + c^2(b-a) - 2abc$
$=$

KUMON

J 56 a Factorization 5

Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	–	1	–	2~

Factor the following expressions.

(1) $x^2(b-c)+b^2(c+x)-c^2(x+b)$
$=$

(2) $x(b^2-c^2)+b(c^2-x^2)+c(x^2-b^2)$
$=$

(3) $x^2(y+z)+y^2(z-x)+z^2(y-x)-2xyz$
$=$

J 56 b

(4) $(x+y+z)(yz+zx+xy)-xyz$
$=$

(5) $a^2b-a^2c-ac^2-ab^2-b^2c+bc^2+2abc$
$=$

J 57 a Factorization 5

Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	–	–	–	1~

Factor the expression by using the two different methods shown in the examples.

Ex. $x^2 - y^2 + 2yz - 2zx - 4x + 2y + 2z + 3$

Arrange the expression in standard polynomial form, with x as the variable.

$x^2 - y^2 + 2yz - 2zx - 4x + 2y + 2z + 3$

$= x^2 + (-2z - 4)x - [y^2 - (2z + 2)y - (2z + 3)]$

Set up the expression as a polynomial in standard form, where x is the variable.

$= x^2 + (-2z - 4)x - (y + 1)[y - (2z + 3)]$

$= [x - (y + 1)][x + (y - 2z - 3)]$

$= (x - y - 1)(x + y - 2z - 3)$

(1) $x^2 - y^2 + 2zx + 2yz + 2y - 2z - 1$

Begin by arranging the expression in standard polynomial form, with x as the variable.

$x^2 - y^2 + 2zx + 2yz + 2y - 2z - 1$

$=$

J 57 b

Ex. $x^2 - y^2 + 2yz - 2zx - 4x + 2y + 2z + 3$

Arrange the expression in standard polynomial form, with z as the variable.

$$x^2 - y^2 + 2yz - 2zx - 4x + 2y + 2z + 3$$

Set up the expression as a polynomial in standard form, where z is the variable.

$$= 2(y - x + 1)z + (x^2 - y^2 - 4x + 2y + 3)$$

$$= 2(y - x + 1)z + [x^2 - 4x - (y^2 - 2y - 3)]$$

$$= 2(y - x + 1)z + [x^2 - 4x - (y - 3)(y + 1)]$$

$$= 2(y - x + 1)z + [x + (y - 3)][x - (y + 1)]$$

$$= -2(x - y - 1)z + (x + y - 3)(x - y - 1)$$

Factor out the common factor $(x - y - 1)$.

$$= (x - y - 1)(x + y - 2z - 3)$$

(2) $x^2 - y^2 + 2zx + 2yz + 2y - 2z - 1$

Begin by arranging the expression in standard polynomial form, with z as the variable.

$x^2 - y^2 + 2zx + 2yz + 2y - 2z - 1$

$=$

J 58 a

Factorization 5

Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	–	–	–	1~

1. $ab + 2ac + 3b^2 + 6bc - 5a - 13b + 4c - 10$

Factor the expression. Begin by arranging it as follows:

In exercise (1), arrange it in standard polynomial form with a as the variable.

In exercise (2), arrange it in standard polynomial form with b as the variable.

(1) With a as the variable,

$ab + 2ac + 3b^2 + 6bc - 5a - 13b + 4c - 10$
$=$

(2) With b as the variable,

$ab + 2ac + 3b^2 + 6bc - 5a - 13b + 4c - 10$
$=$

J 58 b

2. $a^2 + 3b^2 + 4ab + 2ac + 6bc - 4b + 4c - 4$

Factor the polynomial using a, b, or c as the variable.

KUMON

J 59 a

Factorization 5

Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	–	–	–	1~

Factor the following expressions.

(1) $x^3+(2a+1)x^2+(a^2+2a-1)x+(a^2-1)$

$=$

Hint: Arrange in standard polynomial form, with a as the variable.

(2) $ax^2 - a^3 - a^2b + ab^2 + b^3 - bx^2$

$=$

KUMON

J 60 a

Factorization 5

Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	–	–	1	2~

Factor the following expressions.

(1) $x^2 - y^2 + yz - zx - 4x + 2y + z + 3$
$=$

(2) $ac^2 - a^3 - a^2b + ab^2 + b^3 - bc^2$
$=$

J 60 b

(3) $(a+b-c)(ab-bc-ca)+abc$
$=$

(4) $(xy-1)(x-1)(y+1)-xy$
$=$